PERGAMON

International Journal of Solids and Structures 38 (2001) 9201–9208

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Multiple collinear cracks in a piezoelectric material

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Received 3 October 2000; in revised form 4 March 2001

Abstract

In this paper, an exact solution of multiple collinear cracks in piezoelectric material is obtained. The permittivity of air (environment) is considered. Two cases have been studied. In the first case, the permittivity of air is far less than that of piezoelectric material. Therefore, the electric induction in the air is negligible. In the second case, the permittivity of air is comparable with that of piezoelectric material. The problem is deduced into Riemann–Hilbert problem and solved. By the way, the electric boundary conditions are discussed. This result demonstrates that the consideration of air in crack gap reduces the stress intensity factor. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Piezoelectric; Analytic; Multiple cracks; Permittivity; Boundary condition

1. Introduction

From electric engineering to information technology and from ferroelectrics to piezoelectric materials, the electric fracture mechanics is very important. It has been 23 years since Cherepanov (1977) introduced J-integral in the electromagnetic field.

Although many experts Gao et al. (1997); Hao et al. (1996); Sosa (1992); Suo (1991); Suo et al. (1992); Zhang and Tong (1996); Zhang et al. (1997); Zhong et al. (1997); have studied electric fracture mechanics, there are still arguments about electric boundary conditions at crack surfaces. Some authors as Parton (1976), Mikhailov and Parton (1990) considered the following (it is supposed that the crack is located on the Ox_1-axis):

$$D_2^+ = D_2^-, \quad \phi^- = \phi^+, \tag{1}$$

where D_2 is the normal component of electric displacement component and ϕ is the electric potential. The boundary condition (1) has been argued by Pak (1990).

Others have supposed that air (for convenience, the air is used to replace environment because they do not have any mathematical difference) enters when a crack becomes a gap. The permittivity of air is far less than those of piezoelectric materials, so the electric boundary condition can be written as Deeg (1980)

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0020-7683/01/$ - see front matter
PII: S0020-7683(01)00069-5

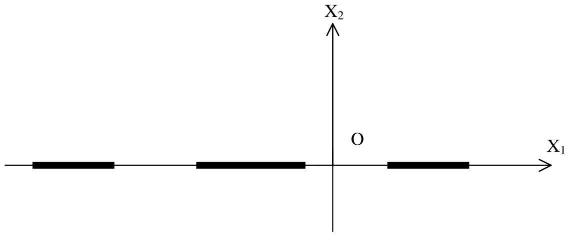

Fig. 1. Multiple cracks.

$$D_2^+ = D_2^- = 0. \tag{2}$$

Some times, the crack gap is full of conduction fluid as McMeeking (1987). One has

$$E_1 = 0 \ \text{ or } \ \phi = \text{const}. \tag{3}$$

Neither Eq. (1) nor Eq. (2) can avoid their incompletion. If the permittivity of air is not quite smaller than that of the solid, the electric induction in the air (environment) cannot be neglected.

Strictly, the electric boundary condition on crack surfaces can be written in the form (if there is no charge in the crack gap and the large deformation is not considered):

$$[D_2] = 0, \ \ [\varphi] = 0, \ \ \Delta\varphi^i = 0, \tag{4}$$

where $[D_2]$ is the jump of electric displacement component between crack surface and air in deformed crack gap, $[\varphi]$ is the jump of potential φ at same place, φ^i is the φ in deformed crack gap and $\Delta\varphi^i = 0$ is $((\partial^2/\partial x_1^2) + (\partial^2/\partial x_2^2))\varphi^i = 0$.

Thus, one has to solve Laplace equation in the deformed crack gap. It is too difficult to deal with.

Considering the gap is very small after deformation, Hao et al. (1994) have used the linear change of φ along the normal of crack surface to replace the rigorous solution of Laplace equation. In the gap, E_2 (for small deformation case, E_n is replaced by E_2) becomes $-(\varphi^+ - \varphi^-)/(u_2^+ - u_2^-)$, and u_2 is the displacement component.

Considering $D_2 = \varepsilon_a E_2$ in air, they obtain

$$D_2 = -\varepsilon_a(\varphi^+ - \varphi^-)/(u_2^+ - u_2^-), \tag{5}$$

where ε_a is the permittivity of air. Since in the gap, E_2 becomes a constant along the normal, D_2 is also a constant along the normal. Then, they obtain

$$D_2^+ = D_2^- = -\varepsilon_a(\varphi^+ - \varphi^-)/(u_2^+ - u_2^-). \tag{6}$$

It is apparent that Eq. (5) will be reduced to second one of Eq. (1) when $u_2^+ - u_2^- = 0$, and to Eq. (2) under the condition $\varepsilon_a = 0$.

Although this boundary condition has considered the air in the crack gap, there is no solution related to this boundary condition except one crack problem. In this paper, the problem of multiple collinear cracks is considered (Fig. 1). Two cases have been dealt with. The first is of boundary condition (2) and the second is of condition (6).

2. Basic equations

In accordance with Lekhnitskii (1981) and Savin (1961), the constitutive equation is rewritten in the form:

$$\varepsilon_i = a_{ij}\sigma_j + g_{ki}D_k, \qquad E_l = -g_{lj}\sigma_j + \delta_{lk}D_k, \tag{7}$$

where the means of $\varepsilon_{i\ldots}$ can be understood by referring to Hao et al. (1994).

When the components of stresses and electric displacement are independent of x_3, there are four complex functions $f_j(z_j)$ ($z_j = x_1 + \mu_j x_2$). The four constants μ_j are complex with positive imaginary part. The stresses and electric displacements can be represented by the linear combination of these functions:

$$\sigma_i = 2\,\mathrm{Re}\sum_{j=1}^{4} k_{ij}f_j''(z_j), \quad D_\mathrm{a} = 2\,\mathrm{Re}\sum_{j=1}^{4} d_{aj}f_j''(z_j) \qquad k_{1j} = \mu_j \;\; k_{2j} = 1, \;\; k_{5j} = a_j\mu_j \;\; k_{4j} = -a_j, \;\; k_{6j} = -\mu_j,$$
$$\mathrm{d}_{1j} = b_j\mu_j, \;\; d_{2j} = -b_j, \tag{8}$$

where a_j, b_j and other coefficients in later formulae of this part can be referred to Hao et al. (1994).

It is not difficult to express the displacement and potential, such as

$$\varphi^+ - \varphi^- = 2\,\mathrm{Re}\sum_{j=1}^{4}(h_{1r}k_{rj} - \xi_{1\alpha}d_{\alpha j})[f_j'(x_1)^+ - f_j'(x_1)^-], \tag{9}$$

$$u_2^+ - u_2^- = 2\,\mathrm{Re}\sum_{j=1}^{4}(\beta_{2r}k_{rj} + \eta_{2a}d_{aj})[f_j'(x_1)^+ - f_j'(x_1)^-]/\mu_j, \tag{10}$$

where $r = 1, 2, 4, 5, 6$ and $\alpha = 1, 2$.

As the problem of multiply connected body is studied, the single value requirements of displacement and potential have to be considered. They are that the value $f_j'(x_1)^+ - f_j'(x_1)^-$ must be zero at any point on Ox_1 except crack surfaces. For these requirements, one chooses n points on Ox_1 except crack surfaces. At the n chosen points, there must be

$$f_j'(x_1)^+ - f_j'(x_1)^- = 0. \tag{11}$$

It is allowed that between two neighbouring cracks, only one chosen point or the beginning point o of integral $\int_\mathrm{o}^{x_1} f_j''(x_1)\,\mathrm{d}x_1$ is located.

3. Boundary conditions

Only the cracks on Ox_1 are considered; therefore, on crack surface, it is supposed that

$$\sigma_2 = \sigma_4 = \sigma_6 = 0 \tag{12}$$

$$D_2^+ = D_2^- = 0 \tag{13}$$

or

$$D_2^+ = D_2^- = -\varepsilon_\mathrm{a}(\varphi^+ - \varphi^-)/(u_2^+ - u_2^-). \tag{14}$$

At infinity

$$\sigma_i = \sigma_i^\infty, \quad D_i = D_i^\infty. \tag{15}$$

Later, the case $D_2^+ = D_2^- = m$ is considered.

It is solved with the substituting method. Firstly, let $D_2 = m$ and $\sigma_i = 0$ in all plane. Therefore, a constant solution is obtained. In this solution, the cracks are not open. Secondly, The electric displacement is $D_2^\infty - m$ at infinity and zero at crack surfaces. Add this solution and constant solution, one will have the solution of the original problem.

4. Solution to the problem

Using the boundary conditions at infinite ∞ and points on Ox_1, this problem is deduced to Riemann–Hilbert problem.

Firstly, the boundary condition (2) ($D_2^+ = D_2^- = 0$ on crack surfaces) is considered and then, the boundary condition (6) are dealt with.

The boundary condition (2) and the mechanical boundary conditions are

$$D_2 = \sigma_2 = \sigma_4 = \sigma_6 = 0 \quad \text{on crack surfaces.} \tag{16}$$

For the linear combination of the four complex variable functions, Eq. (16) becomes

$$2\,\mathrm{Re}\sum_{j=1}^{4} k_{ij} f_j''(x_1) = 0, \qquad 2\,\mathrm{Re}\sum_{j=1}^{4} d_{2j} f_j''(x_1) = 0 \quad \text{on crack surfaces} \tag{17}$$

where $j = 1$–4, $i = 2, 4, 6$.

Using the new notations l_{ij}, these conditions $D_2 = \sigma_2 = \sigma_4 = \sigma_6 = 0$ also can be rewritten into the form:

$$\sum_{j=1}^{4} l_{ij} f_j''(x_1) + \sum_{j=1}^{4} \bar{l}_{ij} \bar{f}_j''(\bar{x}_1) = 0 \quad \text{on crack upper and lower surfaces} \tag{18}$$

where $j = 1$–4, $i = 1, 2, 3$, $l_{ij} = k_{2i,j}$, $i = 4$, $l_{ij} = d_{2j}$.

From relation (18) $D_2 = \sigma_2 = \sigma_4 = \sigma_6 = 0$ and the continuous condition in the other part of Ox_1 (here, the body is continuous and the functions are also continuous), the function $\sum_{j=1}^{4} l_{ij} f_j''(z) - \sum_{j=1}^{4} \bar{l}_{ij} \bar{f}_j''(z)$ is analytical in whole plane. Because one can consider infinite (∞) as a point and the value of the combination $\sum_{j=1}^{4} l_{ij} f_j''(z) - \sum_{j=1}^{4} \bar{l}_{ij} \bar{f}_j''(z)$ is not infinitely large, it must be a constant in all plane. Considering it on Ox_1, one supposes this constant being imaginary $2\mathrm{i}v_i$.

$$\sum_{j=1}^{4} l_{ij} f_j''(z) - \sum_{j=1}^{4} \bar{l}_{ij} \bar{f}_j''(z) = 2\mathrm{i}v_i. \tag{19}$$

Substituting Eq. (19) into Eq. (18), one has

$$\sum_{j=1}^{4} l_{ij} f_j''(x_1)^+ + \sum_{j=1}^{4} l_{ij} f_j''(x_1)^- = 2\mathrm{i}v_i \quad \text{on crack surfaces.} \tag{20}$$

At infinite point,

$$\sum_{j=1}^{4} l_{ij} f_j''(z) = e_i + \mathrm{i}v_i, \tag{21}$$

where e_i is determined by the remote boundary condition (15) and v_i by Eq. (19).

By Muskhelishvili (1963), one obtains

$$\sum_{j=1}^{4} l_{ij} f_j''(z) = e_i + \mathrm{i}f_i + e_\mathrm{i}[Q(z) - 1], \tag{22}$$

$$Q(z) = [z^n + c_1 z^{n-1} + \cdots + c_{n-1} z + c_n] \Bigg/ \left[\prod_{k=1}^{n} (z - a_k)(z - b_k)\right]^{1/2}, \tag{23}$$

where $c_1, c_2, \ldots, c_{n-1}, c_n$ are defined by single value requirements of displacements and potential and a_k and b_k are the two tips of the kth crack. They will be discussed.

The function $\sum_{j=1}^{4} l_{ij} f_j''(z_j)$ will be found by the known $\sum_{j=1}^{4} l_{ij} f_j''(z)$. The unknown imaginary constants $\mathrm{i}v_i$ are no use for us and will not be discussed.

Then, using linear algebra method, one can find functions $f_j''(z_j)$ as:

$$f_i''(z_i) = \sum_{j=1}^{4} x_{ij}\{e_j + \mathrm{i}f_j + e_j[Q(z_i) - 1]\}, \tag{24}$$

where x_{ij} is determined by linear algebra method.

In order to satisfy the requirement of single value, the functions $f_j'(x_1)^+ - f_j'(x_1)^-$ must be dealt with. For these reasons, one has to find the function $\int_0^{x_1} Q(u)\,\mathrm{d}u = P(x_1)$ where point o is located on $\mathrm{O}x_1$ (not on the crack surfaces). As mentioned above, any point to check the requirement of single value cannot be located between the same two cracks with point o. One takes the integral path upon the cracks for $f_j'(x_1)^+$ and below the cracks for $f_j'(x_1)^-$. It is found that

$$\begin{aligned} f_i'(x_1)^+ - f_i'(x_1)^- &= \left[\sum_{j=1}^{4} x_{ij} e_j P(x_1)\right]^- - \left[\sum_{j=1}^{4} x_{ij} e_j P(x_1)\right]^+ = \sum_{j=1}^{4} x_{ij} e_j [P(x_1)^- - P(x_1)^+] \\ &= A_i[P(x_1)^- - P(x_1)^+], \end{aligned} \tag{25}$$

where $A_i = \sum_{j=1}^{4} x_{ij} e_j$.

When constant A_i is not equal to zero, the requirement of single value is

$$P(x_1)^- - P(x_1)^+ = 0. \tag{26}$$

The location of the point x_1 was discussed above.

When the single value requirement is satisfied, the functions $f_i''(z_i)$ are known. The case for boundary condition (2) is solved. The detail will be discussed in numerical example.

Now, the electric boundary condition (6) are concerned with. It is only considered that at infinite, electric displacement component is $D_2^\infty - D_2^+$ where D_2^+ is unknown D_2 of upper surface in crack gap. There must be

$$\begin{aligned} D_2^+ &= -\varepsilon_\mathrm{a}(\varphi^+ - \varphi^-)/(u_2^+ - u_2^-) \\ &= -\varepsilon_\mathrm{a}\mathrm{Re}\sum_{j=1}^{4}(h_{1r}\mathrm{k}_{rj} - \xi_{1\alpha}d_{\alpha j})A_j[P(x_1)^- - P(x_1)^+] \Big/ \mathrm{Re}\sum_{j=1}^{4}(\beta_{2r}k_{rj} + \eta_{2\alpha}d_{\alpha j})A_j[P(x_1)^- - P(x_1)^+]/\mu_j, \end{aligned} \tag{27}$$

where $r = 1, 2, 4, 5, 6$ and $\alpha = 1, 2$.

On crack surfaces, one can find that the function $[P(x_1)^+ - P(x_1)^-]$ is imaginary. Then, Eq. (27) becomes

$$D_2^+ = -\varepsilon_\mathrm{a}\left[\mathrm{Im}\sum_{j=1}^{4}(h_{1\mathrm{r}}k_{rj} - \xi_{1\alpha}d_{\alpha_j})A_j\right] \Big/ \left[\mathrm{Im}\sum_{j=1}^{4}(\beta_{2r}k_{rj} + \eta_{2\alpha}d_{\alpha j})A_j/\mu_j\right]. \tag{28}$$

It is changed with σ_i^∞, $D_i^\infty(D_2^\infty - D_2^+)$ and material constants but not with coordinate. Eq. (28) can be rewritten into this form:

$$D_2^+ = (n + pD_2^+)/(q + tD_2^+), \tag{29}$$

where the coefficients n, p, q, t are determined in Eq. (30) i.e. by σ_i^∞, D_i^∞ and material constants. Eq. (29) becomes a quadratic equation of D_2^+ and can be solved.

When D_2^+ is found, let it be the constant m and one can solve this question as mentioned above.

5. Numerical example

In this section it is assumed that the piezoelectric materials is a PZT-4 ceramic with material constants that can be found in Berlincourt et al. (1964).

The plane strain case of a transversely isotropic material is only considered. As Sosa (1991), the isotropic plane is the Oxy plane. The Oxz plane is taken into consideration (if the Oxy plane is dealt with, there is the antiplane case). For convenience, one renames the coordinates such that $x \to x_1$ and $z \to x_2$.

$$\varepsilon_i = a_{ij}\sigma_j + b_{hi}D_h, \qquad E_f = -b_{fj}\sigma_j + \delta_{ih}D_h, \tag{30}$$

where i, $j = 1$–3, f, $h = 1$–2, $\varepsilon_3 = 2\varepsilon_{xz}$, $\sigma_3 = \sigma_{xz}$, $a_{31} = a_{13} = 0$, $a_{23} = a_{32} = 0$ and $b_{11} = b_{12} = b_{23} = 0$.

The material constants, as Sosa (1991), are

$$\begin{aligned} &a_{11} = 8.205 \times 10^{-12}, \quad a_{12} = -3.144 \times 10^{-12}, \quad a_{22} = 7.495 \times 10^{-12}, \quad a_{33} = 19.30 \times 10^{-12}\ (\mathrm{m^2\,N^{-1}}) \\ &b_{21} = -16.62 \times 10^{-3}, \quad b_{22} = 23.96 \times 10^{-3}, \quad b_{13} = 39.40 \times 10^{-3}\ (\mathrm{m^2\,C^{-1}}) \\ &\delta_{11} = 7.66 \times 10^{7}, \quad \delta_{22} = 9.82 \times 10^{7} (\mathrm{V^2\,N^{-1}}). \end{aligned} \tag{31}$$

As there is no coupling with the antiplane case, only three pairs of functions of generalized complex variables $f_j''(z_j)$ are dealt with. These generalized complex variables ($z_j = x_1 + \mu_j x_2$) are

$$\mu_1 = 1.228i, \quad \mu_2 = 0.203 + 1.067i, \quad \mu_3 = -0.203 + 1.067i. \tag{32}$$

It is supposed that $\sigma_2^\infty$and D_2^∞ do not equal zero at infinite (∞). The f_j'' (x_1) are

$$f_1''(x_1) = [-1.9672\sigma_2^\infty - 0.208 \times 10^{10} D_2^\infty]Q(x_1) + \text{constant} \tag{33}$$

$$f_2''(x_1) = [(1.4836 - 1.8731i)\sigma_2^\infty + (0.1041 + 0.07989i)10^{10} D_2^\infty]Q(x_1) + \text{constant} \tag{34}$$

$$f_3''(x_1) = \left[(1.4836 + 1.8731i)\sigma_2^\infty + (0.1041 - 0.07989i)10^{10} D_2^\infty\right]Q(x_1) + \text{constant}. \tag{35}$$

Two cracks $[-a, -b]$ and $[b, a]$ are considered. One has

$$Q(x_1) = [x_1^2 - a^2E(k)/F(k)]/[(x_1^2 - a^2)(x_1^2 - b^2)]^{1/2}, \tag{36}$$

where $k = (a^2 - b^2)^{1/2}/a$, $F(k) = \int_0^{\pi/2}(1 - k^2\sin^2\theta)^{-1/2}\,d\theta$ and $E(k) = \int_0^{\pi/2}(1 - k^2\sin^2\theta)^{1/2}\,d\theta$.

By the way, the SIF is dealt with. For convenience, only $\sigma_2^\infty$and D_2^∞ do not equal zero.

$$\begin{aligned} K_{1a} &= \lim_{x_1\to a}\sqrt{2\pi(x-a)}\sigma_2 = \lim_{x_1\to a}\sqrt{2\pi(x-a)}\sum_{j=1}^{3} f_j''(x_1) = \lim_{x_1\to a}\sqrt{2\pi(x-a)}\sigma_2^\infty Q(x_1) \\ &= \sigma_2^\infty \lim_{x_1\to a}\sqrt{2\pi}[x_1^2 - a^2E(k)/F(k)]/[(x_1+a)(x_1^2 - b^2)]^{1/2} \\ &= \sigma_2^\infty\sqrt{\pi}a^{3/2}[1 - E(k)/F(k)]/(a^2 - b^2)^{1/2}, \quad K_{1-a} = \lim_{x_1\to -a}\sqrt{2\pi(-a-x)}\sigma_2 = K_{1a}, \end{aligned} \tag{37}$$

$$\begin{aligned} K_{1b} &= \lim_{x_1\to b}\sqrt{2\pi(b-x)}\sigma_2 = \lim_{x_1\to b}\sqrt{2\pi(b-x)}\sum_{j=1}^{3} f_j''(x_1) = \lim_{x_1\to b}\sqrt{2\pi(b-x)}\sigma_2^\infty Q(x_1) \\ &= \sigma_2^\infty \lim_{x_1\to b}\sqrt{2\pi}(-i)[x_1^2 - a^2E(k)/F(k)]/[(x_1+b)(x_1^2 - a^2)]^{1/2} \\ &= \sigma_2^\infty\sqrt{\pi}b^{-1/2}[a^2E(k)/F(k) - b^2]/(a^2 - b^2)^{1/2}, \quad K_{1-b} = \lim_{x_1\to -b}\sqrt{2\pi(x+b)}\sigma_2 = K_{1b}. \end{aligned} \tag{38}$$

For one crack case, $b = 0$; $k = 1$; $E(k)/F(k) = 0$ and $K_{1a} = \sigma_2^\infty\sqrt{a\pi} = K_{1-a}$.

By the same way, one has

$$K_{4a} = (D_2^\infty - D_2^+)(\pi)^{1/2} a^{3/2} [1 - E(k)/F(k)]/(a^2 - b^2)^{1/2}, \tag{39}$$

$$K_{4b} = (D_2^\infty - D_2^+)(\pi)^{1/2} b^{-1/2} [a^2 E(k)/F(k) - b^2]/(a^2 - b^2)^{1/2}. \tag{40}$$

It is nature that for boundary condition (2) (impermeable case), it is obtained that

$$K_{4a} = D_2^\infty (\pi)^{1/2} a^{3/2} [1 - E(k)/F(k)]/(a^2 - b^2)^{1/2}, \tag{41}$$

$$K_{4b} = D_2^\infty (\pi)^{1/2} b^{-1/2} [a^2 E(k)/F(k) - b^2]/(a^2 - b^2)^{1/2}. \tag{42}$$

From Eqs. (39)–(42), the ratio of the stress intensity factor K_4 of permeable (considering air) case to impermeable case is $(D_2^\infty - D_2^+)/D_2^\infty$.

Now, an example for the component of electric displacement in deformed crack gap D_2^+ is given.

It is assumed that $D_2^\infty = 10^{-2}\ \mathrm{C\,m^{-2}}$ and $D_2^+ = -\varepsilon_a(\varphi^+ - \varphi^-)/(u_2^+ - u_2^-)$. One obtains

$$\sigma_2^\infty = 10^6\ \mathrm{Nm^{-2}},\quad D_2^+ = 0.96 D_2^\infty,\quad \sigma_2^\infty = 2\times 10^6\ \mathrm{Nm^{-2}},\quad D_2^+ = 0.93 D_2^\infty,$$
$$\sigma_2^\infty = 10^7\ \mathrm{Nm^{-2}},\quad D_2^+ = 0.76 D_2^\infty,\quad \sigma_2^\infty = 2\times 10^7\ \mathrm{Nm^{-2}},\quad D_2^+ = 0.55 D_2^\infty,\quad \sigma_2^\infty = 10^8\ \mathrm{Nm^{-2}},$$
$$D_2^+ = 0.46 D_2^\infty.$$

It is apparent that the higher the component of stress tensor σ_2^∞ is, the less the component of electric displacement at deformed crack surfaces D_2^+ is. The consideration of air in the gap reduces the stress intensity factor K_4.

6. Concluding remarks

An exact solution of collinear multiple cracks in piezoelectric material has been solved by means of complex variable theory. Besides impermeable cases, the permittivity of air (environment) is also dealt with. Further study must focus on the more general method to consider the permittivity of air (environment). It is necessary to discuss the large deformation, as Hao (1990).

Acknowledgements

Thanks to Prof. Shen Ziyuan for his valuable help on English. This work is supported by the National Natural Science Foundation of China.

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